

Engineering Maths First Aid Kit

8.2

Table of derivatives

Introduction

This leaflet provides a table of common functions and their derivatives.

1. The table of derivatives

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k , any constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n , any constant n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\ln x = \log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\sin kx$	$k \cos kx$
$\cos x$	$-\sin x$
$\cos kx$	$-k \sin kx$
$\tan x = \frac{\sin x}{\cos x}$	$\sec^2 x$
$\tan kx$	$k \sec^2 kx$
$\operatorname{cosec} x = \frac{1}{\sin x}$	$-\operatorname{cosec} x \cot x$
$\sec x = \frac{1}{\cos x}$	$\sec x \tan x$
$\cot x = \frac{\cos x}{\sin x}$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$
$\coth x$	$-\operatorname{cosech}^2 x$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$



Exercises

1. In each case, use the table of derivatives to write down $\frac{dy}{dx}$.

- a) $y = 8$
- b) $y = -2$
- c) $y = 0$
- d) $y = x$
- e) $y = x^5$
- f) $y = x^7$
- g) $y = x^{-3}$
- h) $y = x^{1/2}$
- i) $y = x^{-1/2}$
- j) $y = \sin x$
- k) $y = \cos x$
- l) $y = \sin 4x$
- m) $y = \cos \frac{1}{2}x$
- n) $y = e^{4x}$
- o) $y = e^x$
- p) $y = e^{-2x}$
- q) $y = e^{-x}$
- r) $y = \ln x$
- s) $y = \log_e x$
- t) $y = \sqrt{x}$
- u) $y = \sqrt[3]{x}$
- v) $y = \frac{1}{\sqrt{x}}$
- w) $y = e^{x/2}$

2. You should be able to use the table when other variables are used. Find $\frac{dy}{dt}$ if

- a) $y = e^{7t}$, b) $y = t^4$, c) $y = t^{-1}$, d) $y = \sin 3t$.

Answers

1. a) 0, b) 0, c) 0, d) 1, e) $5x^4$, f) $7x^6$, g) $-3x^{-4}$, h) $\frac{1}{2}x^{-1/2}$, i) $-\frac{1}{2}x^{-3/2}$, j) $\cos x$,
k) $-\sin x$, l) $4 \cos 4x$, m) $-\frac{1}{2} \sin \frac{1}{2}x$, n) $4e^{4x}$, o) e^x , p) $-2e^{-2x}$, q) $-e^{-x}$, r) $\frac{1}{x}$, s) $\frac{1}{x}$
t) $\frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$, u) $\frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$, v) $-\frac{1}{2}x^{-3/2}$, w) $\frac{1}{2}e^{x/2}$.
2. a) $7e^{7t}$, b) $4t^3$, c) $-\frac{1}{t^2}$, d) $3 \cos 3t$.

